Short Papers

Hysteresis Effects in Microwave Amplifiers and Phase-Locked Oscillators Caused by Amplitude-Dependent Susceptance

BERTIL HANSSON

Abstract—**A possible explanation for a type of experimentally observed amplitude instability for amplifiers and locked oscillators is given. The results obtained are in good agreement qualitatively with measurements on IMPATT-diode amplifiers. The theory shows that the susceptance of the active element has to be amplitude dependent to create the actual type of instability.**

I. Introduction

Stability criteria for a phase-locked negative-conductance oscillator are derived in [1]. The criteria are derived in terms of the total admittance, which is the sum of the admittance of the negative-conductance element and the circuit admittance as seen from this element. The circuit admittance is independent of amplitude. For the amplitude dependence of the admittance of the active element a series expansion was used. This type of expansion had been used earlier by Hines [2]. The stability of phase-locked oscillators also has been examined using describing-function techniques with almost identical results [3], [4].

The stability criteria are also valid for a negative-conductance amplifier and they will be used here to describe a type of instability often found experimentally. Fig. 1 shows some curves of output power versus frequency for an IMPATT-diode amplifier. Similar curves have also been observed by others [5], [6]. The main feature of the curves is that the frequency at which maximum gain occurs moves towards lower frequencies when the input power increases. This effect indicates that the susceptance of the active element increases with increasing RF-amplitude. In some cases, jump and hysteresis effects occur in the P_{out} versus frequency curves. This instability is explained theoretically in the following sections. The hysteresis shows that for some input powers and frequencies we have two stable states with different output powers from the amplifier and different RF voltages across the active element.

II. Stability Equations

We consider a circulator-coupled negative-conductance device with a general coupling circuit. The circuit may be viewed as an active element paralleled with a passive circuit admittance, into which a current related to the input power is injected (see Fig. 1 in [1]). The derivation of the stability criteria and an explanation of the *boundary* and *locus curves* for such a circuit are also given in [1]. The locus curve is a closed curve in a diagram of voltage versus frequency within which all points are unstable. In the same diagram all points below the boundary curve are unstable. This is shown in Fig. 2 where shaded areas correspond to unstable points.

For an amplifier the equation for the boundary curve has no real solutions so the only curve we have to consider is the locus curve. Equation (20) in [1] is the equation for the locus curve

$$G\cdot(G + dG/dV\cdot V) + B\cdot(B + dB/dV\cdot V) = 0 \qquad (1)$$

where G is the total conductance and B the total susceptance of the circuit. G and B are, in general, functions of both RF voltage amplitude V and frequency.

For the amplitude dependence, which is only in the active element, a series expansion is used. The current in the element can be written in a series form

$$i = a_1V_d + a_2V_d^2 + a_3V_d^3 + \cdots.$$

We assume that the voltage V_d is a purely sinusoidal voltage at frequency ω.

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The author is with the Research Laboratory of Electronics, Chalmers University of Technology, Gothenburg, Sweden.

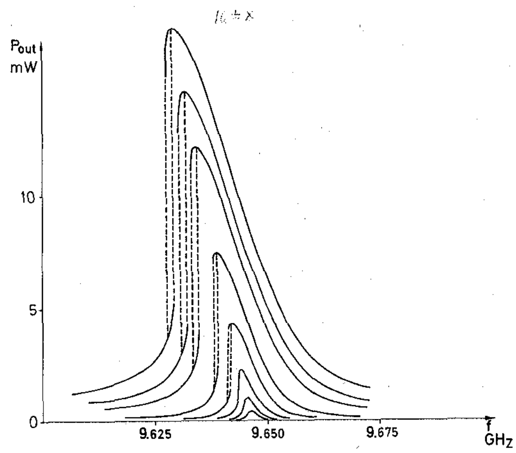

Fig. 1. Experimental output power curves for an IMPATT-diode reflection amplifier with small signal gain 21 dB.

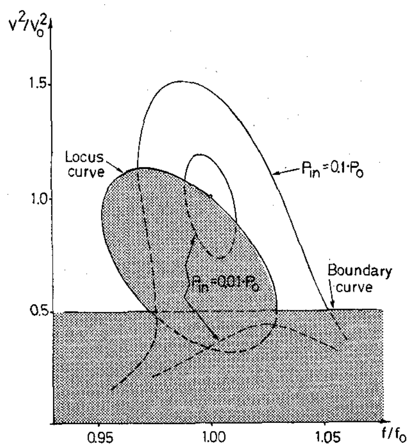

Fig. 2. Voltage amplitude versus frequency with the injected power as a parameter. P_0 is maximum generated power and V_0 the corresponding voltage. A simple parallel resonant circuit with $Q=10$ is used. $G_0/G_c=2.0$ and $B_2/G_2=0.5$.

$$V_d = V\cos\omega t.$$

The fundamental component of the current I is given by [2]

$$I = a_1V + \tfrac{3}{4}a_3V^3 + \tfrac{5}{8}a_5V^5 + \cdots.$$

For the admittance of the active element we get the following series expansion where only the first two terms are taken into account:

$$Y_d = G_0 + G_2\cdot V^2 + j(B_0 + B_2\cdot V^2). \qquad (2)$$

The passive coupling circuit as seen from the active element is described by $Y_c = G_c + j\cdot B_c$. With $G_{0t} = G_0 + G_c$ and $B_{0t} = B_0 + B_c$ the total admittance is

$$Y = G + j\cdot B = G_{0t} + G_2\cdot V^2 + j(B_{0t} + B_2\cdot V^2) \qquad (3)$$

where the coefficients now are only frequency dependent. Introducing this in the equation for the locus curve we obtain

$$(G_{0t} + G_2\cdot V^2)(G_{0t} + 3\cdot G_2\cdot V^2) + (B_{0t} + B_2\cdot V^2)\cdot(B_{0t} + 3\cdot B_2\cdot V^2) = 0 \qquad (4a)$$

or

$$(G_{0t}^2 + B_{0t}^2) + 4\cdot(G_{0t}\cdot G_2 + B_{0t}\cdot B_2)V^2 + 3\cdot(G_2^2 + B_2^2)\cdot V^4 = 0. \qquad (4b)$$

We now introduce $B_{0t}/G_{0t} = a$ and $B_2/G_2 = b$ and get

$$(1 + a^2) + 4\cdot(1 + a\cdot b)\cdot G_{0t}\cdot G_2\cdot V^2 + 3\cdot(1 + b^2)\cdot G_2^2\cdot V^4 = 0. \qquad (4c)$$

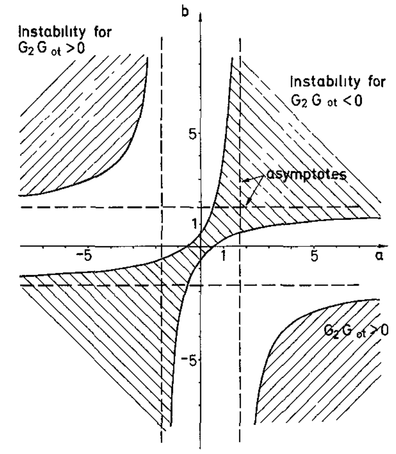

Fig. 3. Stability diagram. The shaded areas are instability regions for $G_2 \cdot G_{0t} > 0$ and $G_2 \cdot G_{0t} < 0$. The border line between shaded and unshaded regions corresponds to the endpoints of the locus curve. Asymptotes are $a, b = \pm\sqrt{3}$.

The conditions for real solutions to (4) are

$$(1 + a \cdot b) \cdot G_{0t} \cdot G_2 < 0 \tag{5a}$$

and

$$a^2b^2 - 3(a^2 + b^2) + 8 \cdot a \cdot b + 1 > 0. \tag{5b}$$

The coefficients a, b, G_{0t}, and G_2 are frequency dependent, and we have real solutions to (4) and a locus curve only in a certain frequency range. For example, this frequency range for the oscillator in Fig. 2 is $f/f_0 = 0.952$–1.028. From (5) we can see that only parameters that influence the existence of a locus curve are a, b, and the sign of $G_{0t} \cdot G_2$. Fig. 3 is a diagram with a and b on the axis. If a and b are within the shaded areas and the sign of $G_2 \cdot G_{0t}$ is that indicated we have a locus curve and a possibility for the type of instability which is connected with the locus curve. For $G_{0t} \cdot G_2 < 0$ we have one shaded area and for $G_{0t} \cdot G_2 > 0$ two different areas.

III. Frequency Range of the Locus Curve

For an amplifier $G_{0t} > 0$ is a necessary condition for small signal stability. G_2 is positive because the negative conductance is decreasing with voltage amplitude. Thus for an amplifier the sign of $G_{0t} \cdot G_2$ is positive. For an IMPATT diode relatively close to the avalanche frequency $B_2 > 0$. A simple single tuned circuit has $dB/d\omega = 2 \cdot C > 0$. With a high Q circuit the parameters of the active element can be assumed to be constant with frequency and the only frequency dependence to take into account is that of B_c. The point corresponding to the center frequency of the amplifier in Fig. 3 is on the axis $a = 0 (B_{0t} = 0)$. When we increase the frequency we move to the right in the diagram along a horizontal line $b = B_2/G_2 = \text{constant}$. Because b is positive with the assumptions given above we move into the shaded area if we decrease the frequency, but only if $b > \sqrt{3}$.

We can also use the same stability diagram for a phase-locked oscillator. For the oscillator the coefficient G_{0t} is negative. The free-running oscillation point is on the symmetry axis $a = b$ in Fig. 3. The area close to this axis is shaded for $G_{0t} \cdot G_2 < 0$ so we have a locus curve extending around the free-running frequency. If we assume small frequency dependence for the parameters of the active element, the only difference is that we move to the left in the diagram with increasing frequency because G_{0t} is negative. If $B_2 > 0$ the locus curve extends more towards lower frequencies. If $B_2/G_2 > \sqrt{3}$ the locus curve will have infinite extension at the low frequency side.

IV. Amplitude Range of Locus Curve

This far we have only considered at which frequencies we have a locus curve. We now want to know something about the amplitude range and therefore determine the amplitudes at the endpoints of the locus curve (as a function of frequency). They are obtained by taking the derivative of (4) with respect to voltage and setting the derivative equal to zero.

$$4 \cdot (G_{0t} \cdot G_2 + B_{0t} \cdot B_2) + 6 \cdot (B_2^2 + G_2^2) V^2 = 0 \tag{6}$$

which gives

$$V^2 = -\frac{2}{3} \frac{G_{0t} \cdot G_2 + B_{0t} \cdot B_2}{G_2^2 + B_2^2} = -\frac{2}{3} \frac{G_{0t}}{G_2} \frac{1 + a \cdot b}{1 + b^2}. \tag{7}$$

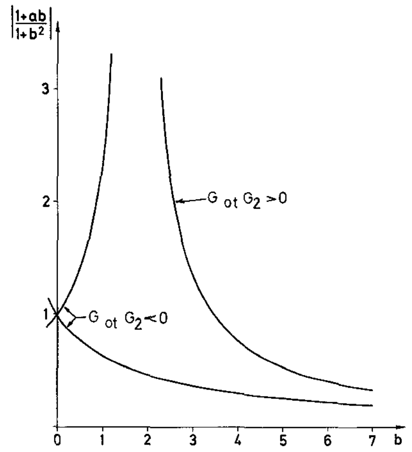

Fig. 4. The factor $(1+ab)/(1+b^2)$ at the border line (in Fig. 3) as a function of b.

This value can also be obtained as a double root to (4). Maximum generated power in the active element occurs when [2]

$$V^2 = V_{\text{opt}}^2 = -G_0/(2 \cdot G_2).$$

For an amplifier the small signal gain is

$$F = (G_c - G_0)/(G_c + G_0) = (G_{0t} - 2G_c)/G_{0t}$$

where G_0 is the small signal conductance of the active element and G_c is the coupling-circuit conductance.

For the amplifier we get

$$V^2/V_{\text{opt}}^2 = -\frac{8/3}{F - 1} \frac{1 + a \cdot b}{1 + b^2} \tag{8}$$

where $(1+ab)/(1+b^2)$ is always negative. The variation of this factor at the endpoint of the locus curve is shown in Fig. 4. We can see that the locus curve moves towards smaller amplitudes with increasing value of $b = B_2/G_2$ and also with increasing gain F.

Turning to the oscillator again we find that the amplitude of the free-running oscillator is $V_0^2 = -G_{0t}/G_2$ (G_{0t} is negative). The amplitude at the endpoints of the locus curve is

$$V^2/V_0^2 = \frac{2}{3} \frac{1 + ab}{1 + b^2}$$

where $(1+ab)/(1+b^2)$ is positive for the oscillator. The amplitude at the endpoints of the locus curve is changing in much the same way as the amplitude of the free-running oscillator. The variation of the factor $(1+ab)/(1+b^2)$ is shown in Fig. 4. Notice that this factor is independents of the sign of b.

V. The Instability Connected with the Locus Curve

Conditions for the existence of a locus curve have only been discussed so far. The type of instability connected with the locus curve should now be examined. Therefore, we look at the output power curves for an amplifier. The equations for calculating input power, diode voltage, output power, and phase have been given earlier. (See for example [1], [2].) Output power versus frequency with input power as a parameter is shown in Fig. 5. The curves have been computed for three different values of small signal gain. The shaded areas are the areas within the locus curves which correspond to unstable points. The output power curves are shown with hysteresis at the locus curve. If we have an amplifier as in Fig. 5(b) and decrease the frequency with $P_{\text{in}} = 0.1 \cdot P_0$, the output power will suddenly drop when we come to the locus curve. If we then increase the frequency we will stay at the low power part until we come to the locus curve again. As is pointed out in [4], the derivative $dP_{\text{out}}/d\omega$ (and also $dV/d\omega$) approaches infinity at the locus curve. The solution to the circuit equations for $P_{\text{in}} = 0.1 \cdot P_0$ is a S-type curve in frequency. The part of the curve within the locus curve is not shown here, but we can see what it looks like for the oscillator in Fig. 2 where the whole curve is drawn. For the amplifier we then have either one stable point or, when the output power curve crosses the locus curve, two stable points and one unstable.

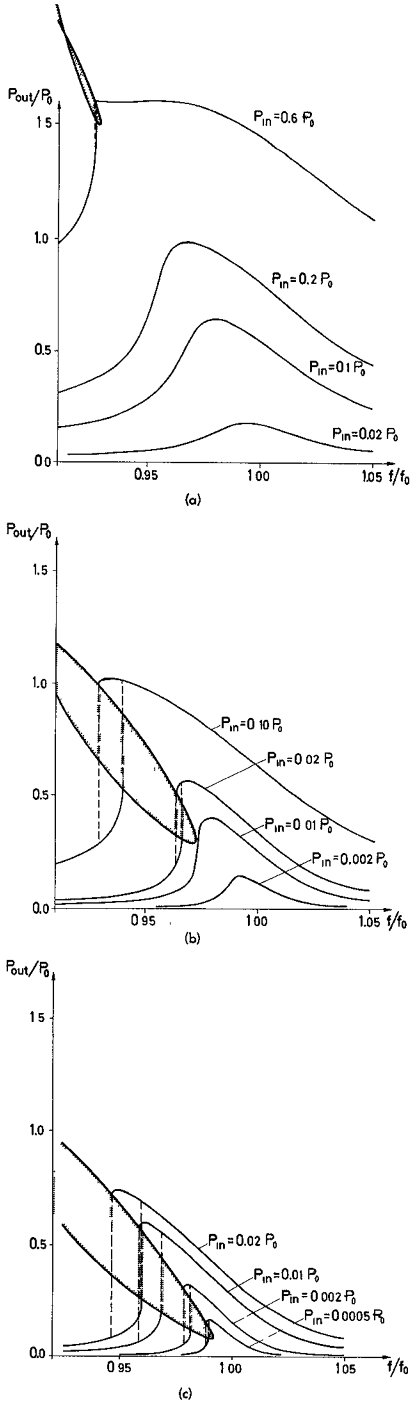

Fig. 5. Output power versus frequency with the input power as a parameter for an amplifier. P_0 is maximum generated power. $B_2/G_2=5$, $Q=10$. (a) Small signal gain 10 dB, $G_0/G_c=-0.519$. (b) Small signal gain 20 dB, $G_0/G_c=-0.818$. (c) Small signal gain 30 dB, $G_0/G_c=-0.939$.

Fig. 5 shows that the locus curve and the connected hysteresis and jump effects move towards the center frequency and towards smaller amplitudes for increasing gain. This can also be concluded from the preceding calculations. Equation (8) directly shows that the amplitude at the endpoint of the locus curve is decreasing with increasing gain F. The endpoint of the locus curve is always at the same value of a, if b is constant. To obtain a constant $a=B_{0t}/G_{0t}$, B_{0t} has to decrease when G_{0t} decreases and the gain increases, which means that the locus curve moves towards the center frequency. It should be noticed that the hysteresis effect described here is created by the amplitude dependence only and has nothing to do with the frequency dependence of either the active element or the circuit.

Turning back to the oscillator in Fig. 2 we look at the curve for $P_{in}=0.1\cdot P_0$. At the frequencies where we have three different states,

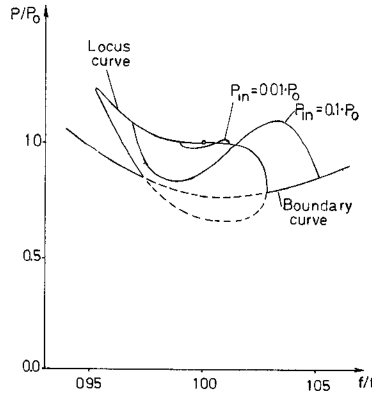

Fig. 6. Output power versus frequency for an oscillator. $G_0/G_c=-2.0$ and $B_2/G_2=0.5$.

only one of these states is stable. One is unstable because it is within the locus curve and one is unstable because it is below the boundary curve. Between $f/f_0=0.952$ and $f/f_0=0.973$ we can have two stable states if one of the points is between the locus and boundary curves in the voltage diagram. This occurs for P_{in} somewhat larger than $0.1\cdot P_0$. Output power curves for the same oscillator are shown in Fig. 6. Here only the stable parts of the curves are drawn. These parts which in the voltage diagram are all above the locus curve are now partly within the locus curve. This is because the active element is optimally loaded for the free-running oscillator and when a signal is injected the element is overdriven and the output power decreases (for some frequencies).

The normal Q value (in contrast with Q_{ext} defined in (27) in [1]) used for both the amplifier and the oscillator is 10. This value was chosen just for computational reasons. For such low Q values the parameters of the active device cannot be assumed to be constant with frequency, but the results can be converted to an arbitrary Q' by using the following transformation

$$Q' = c\cdot Q$$
$$f'/f_0 = (f/f_0 - 1)/c + 1.$$

The model calculations made here are only directly applicable when the frequency dependence of the parameters of the active device can be neglected. They should therefore be used for narrow-band amplifiers and high-Q oscillators. The theory can, however, be used for an arbitrary frequency dependence of the parameters. In Figs. 3 and 4, for example, the frequency is a parameter.

VI. Conclusions

General stability criteria have been applied to a negative-conductance amplifier. The results show that the type of instabilities ovserved for an IMPATT-diode amplifier can be explained by the fact that the diode susceptance has an amplitude dependence. To avoid the hysteresis effects amplitude dependence of the susceptance must be small (if possible $B_2/G_2\leq\sqrt{3}$) or the gain of the amplifier should not be too high. With a broad-band circuit where the parameters of the active element cannot be assumed constant and independent of frequency, the stability diagram (Fig. 3) should be useful to examine instabilities in amplifiers and oscillators. As pointed out before, the hysteresis and jumps discussed in the present paper are only amplitude effects.

References

[1] G. H. B. Hansson and K. I. Lundström, "Stability criteria for phase-locked oscillators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 641–645, Oct. 1972.
[2] M. E. Hines, "Negative-resistance diode power amplification," *IEEE Trans. Electron Devices*, vol. ED-17, pp. 1–8, Jan. 1970.
[3] L. Gustafsson, G. H. B. Hansson, and K. I. Lundström, "On the use of describing functions in the study of nonlinear active microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 402–409, June 1972.
[4] ——, "Maximum phase-locking bandwidth obtainable by injection locking," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-21, pp. 353–355, May 1973.
[5] E. F. Scherer, "Large-signal operation of avalanche-diode amplifiers," *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Microwave Circuit Aspects of Avalanche-Diode and Transferred Electron Devices*), vol. MTT-18, pp. 922–932, Nov. 1970.
[6] H. J. Thaler, G. Ulrich, and G. Weidmann, "Noise in IMPATT diode amplifiers and oscillators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 692–705, Aug. 1971.